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Theme

Numerical method for Fredholm intergro-differential equation by Taylor collocation

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Dedication

I dedicate this work to the most 2 precious and exceptional people in my life who were always there for better or wors ; my parents **Hocine** , **Dalila** to my big supporters during the journey of my studies , my sisters **Halima**, **Imane**, **Amira**, **Samiha**, to those who bring the joy and happiness in my life ;my nieces and nephews , to my God's gift, my soulmates who stand by me through all the ups and downs **Racha**, **Imane** and **Aya** and all my lovely friends I dedicate this work .

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Notations

$\mathbb{R}$:a set of reel numbers.

$\mathbb{C}$:a set of complex numbers.

$\mathbb{N}$:a set of natural numbers .

$\mathbb{R}_n[X]$:a set of polynomial .

IDE: Integro-differential equation .

FIDE: Fredholm Integro-differential equation .

Contents

general introduction

In the last years there has been a growing interest in the Integro-Differential equation , it is an important field in modern Mathematics , where we can find them in many fields which include ; engeneiring , mechanics potential theory electronic ...etc

The Integro-Differential equation of the second kind is an equation in which both differential and integral operators appeared in the same equation , and The Fredholm IDE is characteriesd by the integral boundaries in which are fixed ,

we define it in the next form

$$\begin{cases} y^{(n)}(x) = f(x) + \lambda \int_a^b k(x,t)y(t)dt \\ y^{(n-k)}(0) = \alpha, \quad k = 1, ..., n \end{cases}$$

with k(x,t) is a continuous function , y(x) is the required function and $y^{(n-k)}(0)$ is the initial condition

A Taylor collocation method is presented to solve the FIDE numerically in terms of Taylor polynomial using the Taylor polynomial points

$$y(x) = \sum_{n=0}^{N} \frac{y^{(n)}(c)}{n!}(x_i - c)^n.$$

those equations are usually difficult to solve analytically , So the aim of this thesis is presenting a numerical methode 'Taylor collocation' to find the approach solution for FIDE where this work is organized in the folowing way:

Chapter one : is a remainder about some notions such ;functional, numerical analysis and operators theory , we will mention respectivelly ;the Hilbert, Banach ...(etc)spaces, the Collocation method and taylor seriers finaly the linear ,bounded ...(etc) operators .

Chapter two: is an introductory concepts ,about Differential, Integral and Integro-Differential equations , showing the relationship between IDE and IE.

Chapter three: will be about the method we will use for solving FDIE where we will explain the steps of this method then we will apply it on some examples .

Chapter four : we will solve some equations and find the approach solution using the Matlab programing then we will compare it with the exact solution

Chapter 1

Remainders and fundamental notions

In this chapter we will remaind certain fundamental notions ,and the necessary tools that we need in our thesis concerning the functional space and the operators

1.1 Functional analysis notions

1.1.1 Normed space

Definition 1.1.1. *Let E be a vectorial space in the field* $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$ *we say that E is normed vectorial space if it is endowed a norm* $\|.\|$.

Definition 1.1.2. ***(Norm)*** *We call a norm in the space E any function denoted* $\|.\|$ *defined from E to values in* $\mathbb{R}$*; Such as*

1. $\|x\| = 0 \Longleftrightarrow x = 0$
2. $\|\lambda x\| = |\lambda| \, \|x\| \quad for\ all\ x\ \in\ E, and\ \lambda \in \mathbb{K}$
3. $\left\|x + y\right\| \leq \|x\| + \left\|y\right\| \quad for\ all\ x, y \in E$

Theorem 1.1.1. *All normed space is metrizable*

proof

For all x,y $\in E$, we define the function f by $f(x,y) = \left\|x - y\right\|$ we observe that the function is well metric in E because

$$f(x,y) = \left\|x - y\right\| = 0 \Rightarrow\ x - y = 0$$

Hence

$$x = y$$

it's obvious to see that f(x,y) is symmetrical

$$\begin{aligned} f(x,y) &= \left\|x-y\right\| \\ &= \left\|y-x\right\| = f(y,x) \end{aligned}$$

For the triangle inequality , we write

$$\begin{aligned} f(x,y) = \left\|x-y\right\| &= \left\|(x-z)+(z-y)\right\| \\ &\leq \|x-z\| + \left\|z-y\right\| \\ &= f(x,z)+f(z,y) \end{aligned}$$

1.1.2 Banach space

Definition 1.1.3. *We call Banach space $(E,\|.\|)$ all normed vector and completed for the distance deducted from its norm .*

Definition 1.1.4. *(**Completed space**) Normed space $(E,\|.\|)$ is said completed if all Cauchy sequence x_n with element of E is a convergent sequence in E*

$$\forall \epsilon > 0, \exists N_\epsilon \in N, \forall p,q \geq N_\epsilon \Rightarrow \left\|x_p - x_q\right\| < \epsilon$$

Such as

$$\lim_{n \to +\infty} x_n = x$$

Definition 1.1.5. *(**Cauchy sequence**) Let x_n be a sequence of element of normed space $(E,\|.\|)$, we say that the sequence x_n is Cauchy if the following relation is verified*

$$\forall \epsilon > 0, \exists N_\epsilon, \forall p,q \geq N_\epsilon \text{ we have } \left\|x_p - x_q\right\| < \epsilon$$

Theorem 1.1.2. *Let x_n be a cauchy sequence in normed space $(E,\|.\|)$ contains a* *subsequence x_{n_k} converges toward x. Hence the sequence x_n is also convergent toward the same element x*

proof

Let x_n be a Cauchy sequence then it comes

$$\forall \epsilon > 0, \exists N_\epsilon, \forall p,q \geq N_\epsilon \ we\ have\ \left\|x_p - x_q\right\| < \epsilon$$

We particular for $n_k \geq N_\epsilon$, having

$$\forall p, n_k \geq N_\epsilon \ \ we\ have\ \left\|x_p - x_{n_k}\right\| < \epsilon$$

With the convergent of the sequence x_n toward x

$$n_k \geq N_\epsilon \ , \ \left\|x_{n_k} - x\right\| < \epsilon.$$

Where the convergence of the sequence x_n toward x

$$\begin{aligned}\forall p \ , \ n_k \geq N_\epsilon \ , \ \left\|x_p - x\right\| &= \left\|x_p - x + x_{n_k} - x_{n_k}\right\| \\ &\leq \left\|x_p - x_{n_k}\right\| + \left\|x_{n_k} - x\right\| < \epsilon\end{aligned}$$

1.1.3 Hilbert space

Definition 1.1.6. *A Hilbert space is a Banach space which contains an inner product .*

Definition 1.1.7. ***(Inner product)*** *Let E be a vectorial space in $\mathbb{R}$, an inner product in E is a mapping of $E \times E \in \mathbb{R}$, noted $<.,.>$ verifying the following properties : $\forall x, y, z \in E \ , \ \lambda \in \mathbb{R}$*

1. $< x, x > \geq 0$
2. $< x, x >= 0 \ \Rightarrow \ x = 0$
3. $< \lambda x, y >= \lambda < x, y >$

4. $< x, y+z > = < x, y > + < x+z >$

5. $< x, y > = \overline{< x, y >}$

Remark 1.1.1. *1. A vectorial space that endowed an inner product is called an Euclidien space or Prehilbertien space.*

2. The definItion of inner product gives us the following statements

- $< x+y, z > = < x, z > + < y, z >$.
- $< x, \lambda y > = \overline{\lambda} < x, y >$.

3. We can introduce the inner product as a norm defined by :

$$\|x\| = \sqrt{< x, x >}$$

1.2 Numerical notions

1.2.1 Collocation method

In mathematics,the collocation method is a method of numerical resolution of Integral, Differential and Integro-Differential equations . the idea is choosing a finite dimensional space of candidate solution (in the general polynomials up to a certain degree) and a certain number of points in the domain (which called the collocation points) and selecting the solution which satisfied the equation giving to collocation points .

1.2.2 Taylor series

Definition 1.2.1. *Let y(x) be a function with derivatives of all orders in an interval* $[a, b]$ *wich contains an interior point c,*
a Taylor series is a sum of polynomial terms that expressed the derivatives of y(x) ; to

approximate it at the single point $x = c$, the general form of the Taylor series is :

$$y(x) = \sum_{n=0}^{N} \frac{y^{(n)}(c)}{n!}(x-c)^n.$$

Where
$y^{(n)}$ are the Taylor cofficients

Theorem 1.2.1. *For all $n \in \mathbb{N}$, Taylor formula is given by :*

$$y(x) = \sum_{n=0}^{N} \frac{y^{(n)}(c)}{n!}(x-c)^n$$

And let the Taylor polynomial be $X = (x-c)^n$ which form a base of $\mathbb{R}_n[X]$

proof
Let the Taylor polynomial be :

$$X = (x-c)^n \quad n = 0, 1, ..., N \tag{01}$$

So we will demonstrate that (01) form a free familly of $\mathbb{R}^n[X]$ We consider $\lambda \in \mathbb{R}$ n=0,1,...,N such that :

$$\lambda_n X = 0$$

i.e

$$\lambda_0(x-c)^0 + \lambda_1(x-c)^1 + ... + \lambda_N(x-c)^N = 0$$

For $x \neq 0$
So we obtain

$$\lambda_0 = \lambda_1 = ... = \lambda_n = 0$$

Therefore , $y(x) = (x-c)^n$ is a free familly We can say that the Taylor's polynomials are linearly independent .

1.3 Operator's theory

1.3.1 Bounded linear operators

Definition 1.3.1. ***(The Linearity)*** *Let E and F be two normed spaces , an operator A defined in E to F is called linear if and only if :*

- *Additive condition:*

$$\forall \varphi_1, \varphi_2 \in E \text{ ,} we\ have\ A(\varphi_1 + \varphi_2) = A(\varphi_1) + A(\varphi_2)$$

- *Homogeneous condition :*

$$\forall \varphi \in E \text{ ,} \lambda \in K\ we\ have\ A(\lambda\varphi) = \lambda A(\varphi)$$

Definition 1.3.2. ***(Continuity of linear operator)*** *Let E and F be two normed spaces , a linear operator A defined in $G \subset E$ to F is called continious in x_0 of G if :*
For all sequence x_n of G converges toward x_0 , the sequence $A(x_n)$ converges toward $A(x_0)$ that means

$$\lim_{n\to\infty} A(x_n) = A(\lim_{n\to\infty} x_n) = A(x_0)$$

Remark 1.3.1. *Linear operator A is called continious in G if it is continious in each point of the set G*

Theorem 1.3.1. *Let E,F be two normed spaces , a linear operator A defined in a subset $G \subset E$ to F is called continious everywhere in G If it is continious in point x_0 of G*

Definition 1.3.3. ***(Bounded operator)*** *A linear operator A defined in E to F is called bounded if there exists a positive constant $C > 0$ such that :*

$$\|A(x)\|_F \leq C\,\|x\|_E \quad \forall x \in E$$

Theorem 1.3.2. *A linear operator A is continious iff it is bounded*

proof

We suppose that the operator A is bounded , we have

$$\|A(x)\|_F \leq C\,\|x\|_E$$

Or

$$\|A(x) - A(0)\|_F \leq C\,\|x - 0\|_E$$

Where the countinuity of the operator A in 0 such that

$$\lim_{n\to\infty} A(x) = A(0) \;\; where \;\; \lim_{n\to\infty} x = 0$$

Which leads to the countinuity everywhere .

1.3.2 Integral operator

Definition 1.3.4. *We call integral operator all linear operator A defined in a normed space E to normed space F given by the formula :*

$$A\varphi(x) = \int_{G_2} K(x,y)\varphi(y)dy \qquad x \in G_1$$

Where

K(x,y) *:mesurable function defined in mesury set $G_1 \times G_2$ who is called the kernel of integral operator A $\varphi(y)$: is mesurable function defined in G_2*

1.3.3 Differential operator

Definition 1.3.5. *A linear Differential operator in ordre m is defined by*

$$D = \sum_{\alpha=0}^{m} a_\alpha(x) D^\alpha$$

Where the $a_\alpha(x)$ are functions of n variable where they are called coefficients of operator D

Remark 1.3.2. *Differential operator is an acting operator in the deferentiable functions*

- *When the function has single variable ;the differential operator is constructed from ordinary derivates.*

- *When the function has miltiple variable ;the differential operator is constructed form partial derivates .*

When both differential and integral operator appeared in the same equation we call it Integro-differential operator

Chapter 2

Introduction to the theory of integral and integro differential equations

In this chapter we will have a look at the theory of Integral and Integro-Differential equation where we will mention different types of IDE and the relationship between IDE and IE and will show the analytic solution

2.1 Differential equation

Definition 2.1.1. *Differential equation is an equation where its unknown term is a function , which has a relation between this function and its derivative*

$$\frac{d^{(n)}y}{dx^{(n)}} + a_1(x)\frac{d^{(n-1)}y}{dx^{(n-1)}} + ... + a_n(x)y = F(x)$$

Where F(x) is given. With continious coefficients$(a_1, ..., a_n)$ *and initial conditions:*

$$y(a) = q_0 \quad , \; y'(a) = q_1 ...., y^{(n+1)}(a) = q_{n-1}.$$

Example 2.1.1.

$$\begin{cases} y''(x) + 4y'(x) = sin(x) & 0 \le x \le 1 \\ y(0) = 0 \qquad y(1) = 0 \end{cases}$$

2.2 Integral equation

Definition 2.2.1. *We call an Integral equation all functionall equation where the unknown function y(x) appears under the sign of integration, the general form of an* *integral equation with unknown $y(x)$ is given by:*

$$y(x) = f(x) + \int_E k(x,t)y(t)dt$$

Where $f(x), k(x,t)$ are given
E is measure space .

Example 2.2.1. *1.* $f(x) = \int_E k(x,t)y(t)dt, \quad (first\ kind).$

2. $y(x) = f(x) + \lambda \int_E k(x,t)y(t)dt, \quad \lambda \in \mathbb{R} \qquad (second\ kind).$

2.3 Integro-Differential equation

Definition 2.3.1. *The Integro-Differential equation is characterized by the existence of one or more of the derivatives $y'(x), y''(x)...y^{(n)}$outside the Integral side , plus the Integral operator We define the IDE as :*

$$y^{(n)}(x) + a_1 y^{(n-1)}(x)....a_n y(x) = f(x) + \int_E k(x,t)y(t)dt.$$

Where $a_1, a_2, ..a_n$ are constants.
f(x),k(x,t) are known functions.
$y(x)$ is the unknown function,

2.4 Classification of IDE

We classificate the IDE based on their caracteristique such that :

1. Integration domain.
2. Kind of equation .
3. The nature of the function f(x) (homogenic).
4. The liniarity of the function y .

5. The nature of the kernel and singularity.

In our thesis we will mention only the first type wich is **integration domain** (for other types see [3])
We will define 3 forms of integration domain.

2.4.1 Fredholm Integro-Differential equation

The integral boundaries in FDIE are fixed , Which is given in the form :

$$y^{(n)}(x) = f(x) + \lambda \int_a^b k(x,t)y(t)dt$$

2.4.2 Voltera Integro-Differential equation

In the VIDE one of the Integral boundaries is varied Which is given in the form

$$y^{(n)}(x) = f(x) + \lambda \int_0^x k(x,t)(t)dt$$

2.4.3 Fredholm-Voltera Integro-Differential equation

Here both fredholm and voltera inetgral operators are coincide so we have :

$$y^{(n)}(x) = f(x) + \lambda \int_a^b k(x,t)y(t)dt + \lambda \int_0^x k(x,t)y(t)dt.$$

Remark 2.4.1. *We call the IDE is singular if either one or both of the integral boundaries are infinte, The kernel becomes infinte in the voisinage.*

2.5 Converting Fredholm Integro-Differential equation into Fredholm integral equation

Let us consider the foLlowing FIDE

$$\begin{cases} y'(x) = p(x)y(x) + q(x) - \int_0^1 k(x,t)y(t)dt & 0 \leq x \leq 1 \\ y(0) = \alpha \end{cases}$$

Where $p(x), q(x)$ *and* $k(x,t)$ are continious functions, We put :

$$y'(x) = z(x) \tag{1}$$

We integre both sides of (1) w.r.t to t from 0 to x we find

$$y(x) = \alpha + \int_0^x z(t)dt \tag{2}$$

Substituting (1) and (2) in the FIDE above
We get :

$$z(x) = p(x)\left[\alpha + \int_0^x z(t)dt\right] + q(x) - \int_0^1 k(x,t)\left[\alpha + \int_0^t z(s)ds\right]dt$$

We put

$$g(x) = \alpha p(x) + p(x)\int_0^x z(t)dt + q(x) + \alpha\int_0^1 k(x,t)z(t)dt$$

And

$$h(x) = p(x)\int_0^x z(t)dt + \int_0^1 k(x,t)\left(\int_0^t z(s)ds\right)dt$$

Hence

$$z(x) = g(x) + h(x)$$

We have

$$\int_0^1 k(x,t)\left(\int_0^t z(s)ds\right)dt = \int_0^1 \left(\int_0^t k(x,s)ds\right)z(t)dt$$

We put

$$k'(x,t) = \int_t^1 k(x,s)ds$$

We substitute in h(x) we find

$$h(x) = p(x)\int_0^x z(t)dt + \int_0^1 k'(x,t)z(t)dt$$

Hence

$$h(x) = \int_0^1 \left(a(x).H(x,t) + k'(x,t)\right)z(t)dt$$

Where

$$H(x,t) = \begin{cases} 1 & (s-t) \geq 0, \\ 0 & (s-t) < 0. \end{cases}$$

We put

$$R(x,t) = p(x).H(x,t) + k'(x,t).$$

Finaly we get the FID ;

$$z(x) = g(x) + \int_0^1 R(x,t)z(t)dt.$$

2.6 Analytic solution

A real function y(x) is called analytic if it has derivatives of all orders such that Taylor series at any point c in its domain .

The generic form of Taylor series at x=0 can be written as :

$$\sum_{n=0}^{N} a_n x^n$$

For solving Fredholm Integro-Differential equation we will assume that the solution is analytic

Therefore possesses a Taylor series where the coefficients a_n will be determined recurrently

Substuting Taylor series in FIDE we find :

$$\left(\sum_{n=0}^{\infty} a_n x^n\right)^{(n)} = T(f(x)) + \lambda \int_a^b \left(k(x,t)\sum_{n=0}^{\infty} a_n t^n\right)$$

Where T(F(x)) is the Taylor series for f(x)

Example 2.6.1. *Let us consider the folowing FIDE :*

$$\begin{cases} y'(x) = 4 + 4x + \int_{-1}^{1}(1-xt)y(t)dt \qquad 0 \le x\ ,\ t \le 1 \\ y(0) = 1 \end{cases}$$

We substitute the Taylor series form in the equation we find

$$\left(\sum_{n=0}^{\infty} a_n x^n\right)' = 4 + 4x + \int_{-1}^{1} \left((1-xt)\sum_{n=0}^{\infty} a_n t^n\right)$$

This leads to :

$$\sum_{n=1}^{\infty} n a_n x^{n-1} = 4 + 4x + \int_{-1}^{1} \left((1-xt)\sum_{n=0}^{\infty} a_n t^n\right)$$

After integrating the right side of the integral and collecting the coefficients of like powers of x we find :

$$a_1 + 2a_2 x + 3a_3 x^2 + ... = 6 + \frac{2}{3}a_2 + \frac{2}{5}a_4 + \frac{2}{7}a_6 + \frac{2}{9}a_8 + (4 - \frac{2}{3}a_1 - \frac{2}{5}a_3 - \frac{2}{7}a_5 - \frac{2}{9}a_7)x$$

Equating the coefficients of like powers of x in both sides gives

$$a_1 = 6, \quad a_n = 0, n \geq 2$$

Where the exact solution is given by

$$y(x) = 1 + 6x$$

CHAPTER 3

NUMERICAL RESOLUTION FOR FREDHOLM INTEGRO-DIFFERENTIAL EQUATION BY TAYLOR COLLOCATION METHOD

In this chapter , the Taylor Colocation method is developed for FIDE Where we explained the different steps , with an illustrating example

3.1 Description of the method

We have the Fredholm Integro-Differential equation :

$$\sum_{k=0}^{m} p_k(x)y^{(n)}(x) = f(x) + \lambda \int_a^b k(x,t)y(t)dt \qquad \text{(eq:1)}$$

Where the known functions f(x) , $p_k(x)$, k(x,t) are defined on the $a \leq x,\ t \leq b,\ \lambda$ is a real parameter , y(x) is the unknown function We mention that the condition in the most general can be written in the form :

$$\sum_{j=0}^{m-1} \left[a_{ij}y^{(j)}(a) + b_{ij}y^{(j)}(b) + c_{ij}y^{(j)}(c)\right] = \lambda_i; i = 0, 1....m-1 \qquad \text{(eq:2)}$$

Where $a \leq c \leq b$, a_{ij}, b_{ij}, c_{ij} are appropriate constants
We have the truncated Taylor series is

$$y(x) = \sum_{n=0}^{N} \frac{y^{(n)}(c)}{n!}(x-c)^n \qquad \text{(eq:3)}$$

Where $y^{(n)}(c)$ are Taylor coefficients .
We assume that the solution of (eq:1) can be truncated Taylor series as (eq:3) If we suppose that the functions $p_k(x)$ in (eq:1)are defined in $a \leq x \leq b$ and k(x,t) is defined also and if bounded variation in $a \leq x, t \leq b$, that is k(x,t) can be expanded to Taylor series If we put

$$n=0 \Rightarrow y(x)=\frac{y^{(0)}(c)}{0!}(x-c)^0$$
$$n=1 \Rightarrow y(x)=\frac{y^{(1)}(c)}{1!}(x-c)^1$$
$$n=2 \Rightarrow y(x)=\frac{y^{(2)}(c)}{2!}(x-c)^2$$
$$\vdots$$
$$n=N \Rightarrow y(x)=\frac{y^{(N)}(c)}{N!}(x-c)^N$$

We can see that

$$A=\left[y^{(0)}(c) \;\; y^{(1)}(c) \;\; y^{(2)}(c) \;\; ...y^{(N)}(c)\right]^t$$
$$X=\left[(x-c)^0 \;\; (x-c)^1 \;\; (x-c)^2 \;\; ....(x-c)^N\right]$$
$$M_0=\begin{pmatrix} \frac{1}{0!} & 0 & 0 & 0 & ...0 \\ 0 & \frac{1}{1!} & 0 & 0 & ...0 \\ 0 & 0 & \frac{1}{2} & 0 & ...0 \\ 0 & 0 & 0 & \frac{1}{3!} & ...0 \\ 0 & .. & .. & .. & ...\frac{1}{N!} \end{pmatrix}$$

So that we can write the solution in the matrix form such as ;

$$[y(x)]=XM_0A$$

and now we apply the Taylor collocation method,
We define ***the Taylor collocation points*** *by :*

$$x_i=a+i\frac{b-a}{N}; \qquad i=0,1..N; \qquad x_0=a, \quad x_1=b \qquad \text{(eq:4)}$$

We substitute (eq:4) into (eq:1) we find

$$\sum_{k=0}^{m} p_k(x_i)y^{(n)}(x_i) = f(x_i) + \lambda I(x_i) \tag{eq:5}$$

Where

$$I(x_i) = \int_a^b k(x_i,t)y(t)dt.$$

With the same method we did to (eq:3) we can write The matrix form of the system (eq:5)

$$P_0Y^{(0)} + P_1Y^{(1)} + ... + P_mY^{(m)} = \sum_{k=0}^{m} P_kY^{(k)} = F + \lambda I \tag{eq:6}$$

Where

$$P_k = \begin{pmatrix} P_0(x_0) & 0 & \ldots 0 \\ 0 & P_1(x_1) & \ldots 0 \\ \vdots & \vdots & \ddots \vdots \\ 0 & 0 & \ldots P_m(x_N) \end{pmatrix}; F = \begin{pmatrix} f(x_0) \\ f(x_1) \\ f(x_2) \\ \vdots \\ f(x_N) \end{pmatrix}; Y^k = \begin{pmatrix} y^{(0)}(x_0) \\ y^{(1)}(x_1) \\ y^{(2)}(x_2) \\ \vdots \\ y^{(m)}(x_N) \end{pmatrix};$$

$$I = \begin{pmatrix} I(x_0) \\ I(x_1) \\ I(x_2) \\ \vdots \\ I(x_N) \end{pmatrix}$$

We derivate the function (eq:3) w.r.t x we find :

$y'(x) = \frac{y^{(n)}(c)}{n\times(n-1)\times...\times 1} n(x-c)^{n-1}$

$y'(x) = \frac{y^{(n)}(c)}{(n-1)\times...\times 1} (x-c)^{n-1}$

Then we derivate for the second time we find :

$y^{''}(x) = \frac{y^{(n)}(c)}{(n-1)\times(n-2)\times...\times 1} (n-1)(x-c)^{n-2}$

$y^{''}(x) = \frac{y^{(n)}(c)}{(n-2)\times...\times 1} (x-c)^{n-2}$

$\vdots$

Hence we find the kth derivative :

$$y^{(k)}(x) = \sum_{n=k}^{N} \frac{y^{(n)}(c)}{(n-k)!}(x-c)^{n-k} \qquad a \le x \le b$$

Substituting in (eq:4) we find :

$$y^{(k)}(x_i) = \sum_{n=k}^{N} \frac{y^{(n)}(c)}{(n-k)!}(x_i-c)^{n-k} \qquad a \le x \le b$$

With the same method we find the matrix form :

$$[y^{(k)}(x_i)] = CM_kA \qquad (k=0,1,...,N) \tag{3.1}$$

Where

$$C = \begin{pmatrix} X_{x_0} \\ X_{x_1} \\ \vdots \\ X_{x_N} \end{pmatrix} = \begin{pmatrix} (x_0-c)^0 & (x_0-c)^1 & \dots (x_0-c)^N \\ (x_1-c)^0 & (x_1-c)^1 & \dots (x_1-c)^N \\ \vdots & \vdots & \ddots \vdots \\ (x_N-c)^0 & (x_N-c)^1 & \dots (x_N-c)^N \end{pmatrix}$$

$$A = \begin{pmatrix} y^{(0)}(x) \\ y^{(1)}(x) \\ \vdots \\ y^{(n)}(x) \end{pmatrix}$$

$$M_k = \begin{pmatrix} 0 & 0 & \dots \frac{1}{0!} & 0 & \dots 0 \\ 0 & 0 & 0 & \frac{1}{1!} & \dots 0 \\ \vdots & \vdots & \ddots \vdots & \vdots & \ddots \vdots \\ 0 & 0 & \dots 0 & 0 & \dots \frac{1}{(N-k)!} \\ \vdots & \vdots & \ddots \vdots & \vdots & \ddots \vdots \\ 0 & 0 & \dots 0 & 0 & \dots 0 \end{pmatrix}$$

We write the system (eq:6) as :

$$\left(\sum_{k=0}^{m} P_k C M_k\right) A = F + \lambda I \qquad \text{(eq:8)}$$

Now we truncated the kernel k(x,t) to Taylor series (when $x = c$ and $t = c$) in order to find the matrix I

$$k(x,t) = \sum_{n=0}^{N} \frac{k^{(n)}(c)}{n!}(x-c)^n . \sum_{m=0}^{N} \frac{k^{(m)}(c)}{m!}(t-c)^m$$

$$k(x,t) = \int_a^b \frac{k^{(n+m)}(c)}{n!m!}(x-c)^n(t-c)^m dt$$

Hence

$$k_{nm} = \frac{1}{n!m!}\frac{\partial^{n+m}}{\partial x^n \partial x^m}$$

So the formula becomes

$$k(x,t) = \sum_{n=0}^{N}\sum_{m=0}^{N} k_{nm}(x-c)^n(t-c)^m$$

We can write the matrix form of k(x,t) by the formula :

$$[k(x,t)] = XKT^t \qquad \text{(eq:9)}$$

Where

$$K = \begin{pmatrix} k_{00} & k_{01} & \dots k_{0N} \\ k_{10} & k_{11} & \dots k_{1N} \\ \vdots & \vdots & \ddots \\ k_{N0} & K_{N1} & \dots k_{NN} \end{pmatrix}$$

$$X = \begin{pmatrix} (x-c)^0 & (x-c)^1 & \dots (x-c)^N \end{pmatrix} \; ; \; T = \begin{pmatrix} (t-c)^0 & (t-c)^1 & \dots (t-c)^N \end{pmatrix}^t$$

Hence ; we have

$$[y(x)] = XM_0A\,, \quad [y(t)] = TM_0A \qquad \text{(eq:10)}$$

We find the expression of the integral I in the matrix form by :

$$I(x) = \int_a^b (XKT^tTM_0A)dt \qquad \text{(eq:11)}$$

Such that we define :

$$H = [h_{nm}] = \int_a^b T^tTdt$$

$$T^tT = \begin{pmatrix} (t-c)^0 & (t-c)^1 & \dots(t-c)^n \\ (t-c)^1 & (t-c)^2 & \dots(t-c)^{n+1} \\ \vdots & \ddots & \vdots \\ (t-c)^m & \dots & \dots(t-c)^{n+m} \end{pmatrix}$$

So we integre the matrix we find

$$\int_a^b T^tTdt = \begin{pmatrix} (t-c)|_a^b & \frac{1}{2}(t-c)^2|_a^b & \dots\frac{1}{n+1}(t-c)^{n+1}|_a^b \\ \frac{1}{2}(t-c)^2|_a^b & \frac{1}{3}(t-c)^3|_a^b & \dots\frac{1}{n+1}(t-c)^{n+1}|_a^b \\ \vdots & \vdots & \vdots \\ \frac{1}{m+1}(t-c)^{m+1}|_a^b & \frac{1}{m+2}(t-c)^{m+2}|_a^b & \dots\frac{1}{n+m+1}(t-c)^{n+m+1}|_a^b \end{pmatrix}$$

We conclude the general form of h_{nm}

$$h_{nm} = \frac{(b-c)^{n+m+1} - (a-c)^{n+m+1}}{n+m+1}|_{n,m=0,1\dots,N} \qquad \text{(eq:12)}$$

We substitute (eq:12) in (eq:11) we find the final formula of the matrix I

$$I = CKHM_0A \qquad \text{(eq:13)}$$

Finaly we substitute (eq:13) in (eq:6) We have

$$\left(\sum_{k=0}^{m} P_kCM_k - \lambda CKHM_0\right)A = F \qquad \text{(eq:14)}$$

We assume that

$$W = \sum_{k=0}^{m} P_kCM_k - \lambda CKHM_0$$

So (eq:14) becomes

$$WA = F \tag{eq:15}$$

$$W = [w_{ij}] = \sum_{k=0}^{m} P_k C M_k - \lambda CKHM_0$$

So we write the augmented matrix :

$$[W;F] = \begin{pmatrix} w_{00} & w_{01} & \dots w_{0N} & ; f(x_0) \\ w_{10} & w_{11} & \dots w_{1N} & ; f(x_1) \\ \vdots & \vdots & \ddots & \vdots \\ w_{N0} & w_{N1} & \dots w_{NN} & ; f(x_N) \end{pmatrix}$$

As we know that solving the FIDE is by finding the Taylor coefficients that presented in the vector A Such that

$$A = W^{-1} F \tag{eq:16}$$

Now we represent the matrix form of the condition giving in (eq:2) using the truncated Taylor series ; we find

$$\sum_{j=0}^{m-1} \left(a_{ij} \left(\underbrace{\sum_{n=j}^{N} \frac{y^{(n)}(c)}{(n-j)!}(a-c)^{n-j}}_{y^{(j)}(a)} \right) + b_{ij} \left(\underbrace{\sum_{n=j}^{N} \frac{y^{(n)}(c)}{(n-j)!}(b-c)^{n-j}}_{y^{(j)}(b)} \right) \right.$$
$$\left. + c_{ij} \left(\underbrace{\sum_{n=j}^{N} \frac{y^{(n)}(c)}{(n-j)!}(c-c)^{n-j}}_{y^{(j)}(c)} \right) \right) = \lambda_i$$

$$i = 0, 1..., m-1, \qquad a \le c \le b$$

Like the previous method we find the matrix representations of : $y^{(j)}(a)$ *,* $y^{(j)}(b)$ *and* $y^{(j)}(c)$ *as :*

$$[y^{(j)}(a)] = PM_j A$$

$$[y^{(j)}(b)] = QM_jA$$

$$[y^{(j)}(c)] = RM_jA$$

such that

$$P = \left[(a-c)^0 \quad (a-c)^1 \quad \ldots(a-c)^N\right]$$

$$Q = \left[(b-c)^0 \quad (b-c)^1 \quad \ldots(b-c)^N\right]$$

$$R = \left[(c-c)^0 \quad 0 \quad \ldots 0\right]$$

Substituting in the matrix representation of the condition

We find

$$\sum_{j=0}^{m-1}(a_{ij}P + b_{ij}Q + c_{ij}R)M_jA = [\lambda_i]$$

We put

$$U_i = \sum_{j=0}^{m-1}(a_{ij}P + b_{ij}Q + c_{ij}R)M_j \equiv [u_{i0} \quad u_{i1} \quad \ldots u_{iN}]$$

$$i = 0, 1 \ldots, m-1$$

Hence

The matrix form of the condition becomes

$$U_iA = [\lambda_i]$$

So that the augmented matrices of theme are

$$[U_i; \lambda_i] = \left[u_{i0} \quad u_{i1} \quad \ldots u_{iN};\ \lambda_i\right] \qquad \text{(eq:17)}$$

Hence Replacing the m rows matrices of (eq:17) by the last m rows of augmented matrix of [W;F] we get the augmented matrix;

$$[\tilde{W}; \tilde{F}]$$

Where

$$\tilde{W} = \begin{pmatrix} w_{00} & w_{01} & \dots & w_{0N} \\ w_{10} & w_{11} & \dots & w_{1N} \\ \vdots & \vdots & \dots & \vdots \\ w_{N-m,0} & w_{N-m,1} & \dots & w_{N-m,N} \\ u_{00} & u_{01} & \dots & u_{0N} \\ \vdots & \vdots & \dots & \vdots \\ u_{m-1,0} & u_{m-1,1} & \dots & u_{m-1,N} \end{pmatrix}, \tilde{F} = \begin{pmatrix} f(x_0) \\ f(x_1) \\ \vdots \\ f(x_{N-m}) \\ \lambda_0 \\ \vdots \\ \lambda_{m-1} \end{pmatrix}$$

If $\left|\tilde{W}\right| \neq 0$ *we write*

$$A = \tilde{W}^{-1}\tilde{F}$$

Finaly We provide our work by an a example :

3.1.1 Example

Let us consider the FIDE :

$$\begin{cases} y'' + xy' - xy = e^x - 2sin(x) + \int_{-1}^{1} sin(x)e^{-t}y(t)dt & -1 \leq x,\ t \leq 1 \\ y(0) = 1 \qquad y'(0) = 1 \end{cases}$$

Which has the exact solution given by : $y(x) = e^x$

We approximate the solution y(x) using the Taylor polynomial for N=3

$$y(x) = \sum_{n=0}^{3} \frac{y^{(n)}(0)}{n!} x^n$$

We have $a = -1$ *,* $b = 1$*,* $c = 0$*,* $\lambda = 1$*,*

$P_0 = -x, \quad P_1 = x, \quad P_2 = 1$

$f(x) = e^x - 2sin(x), \quad k(x,t) = sin(x)e^{-t}$

We have

$$w = \sum_{k=0}^{2} P_k C M_k - \lambda C K H M_0$$

So that

$$w = P_2CM_2 + P_1CM_1 - P_0CM_0 - CKHM_0$$

And the Collocation points are :

$$x_i = -1 + i\frac{2}{3} \quad i = 0, 1, 2, 3$$

Hence

$$p2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad p0 = p1 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{2!} & 0 \\ 0 & 0 & 0 & \frac{1}{3!} \end{pmatrix}, \quad M_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{2!} \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad M_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$K = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & -1 & \frac{1}{2} & -\frac{1}{6} \\ 0 & 0 & 0 & 0 \\ -\frac{1}{6} & \frac{1}{6} & -\frac{1}{12} & \frac{1}{36} \end{pmatrix}, \quad H = \begin{pmatrix} 2 & 0 & \frac{2}{3} & 0 \\ 0 & \frac{2}{3} & 0 & \frac{2}{5} \\ \frac{2}{3} & 0 & \frac{2}{5} & 0 \\ 0 & \frac{2}{5} & 0 & \frac{2}{7} \end{pmatrix}, \quad C = \begin{pmatrix} 1 & -1 & 1 & -1 \\ 1 & -\frac{1}{3} & \frac{1}{9} & -\frac{1}{27} \\ 1 & \frac{1}{3} & \frac{1}{9} & \frac{1}{27} \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

And we have the augmented matrix of the conditions are

$$\begin{bmatrix} 1\,0\,0\,0\,;1 \\ 0\,1\,0\,0\,;1 \end{bmatrix}$$

After calculating we find

$$W = \begin{pmatrix} \frac{53}{18} & -\frac{47}{18} & \frac{103}{36} & \frac{1307}{756} \\ \frac{53}{486} & -\frac{477}{697} & \frac{609}{479} & -\frac{499}{1319} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

And the matrix form of F:

$$F = \begin{pmatrix} \frac{1372}{669} \\ \frac{462}{337} \\ 1 \\ 1 \end{pmatrix}$$

We have $A = W^{-1}F$

$$A = \begin{pmatrix} 1 \\ 1 \\ \frac{475}{526} \\ \frac{241}{481} \end{pmatrix}$$

Finaly we obtain the approximate solution :

$$y(x) = 1 + x + 0.5003x^2 + 0.1669x^3 \cong e^x$$

Chapter 4

Comparison between the exact and the approach solution

In this chapter we worked on some examples then we compared between their exact and approach solution in tables and strengthen our work by grafics

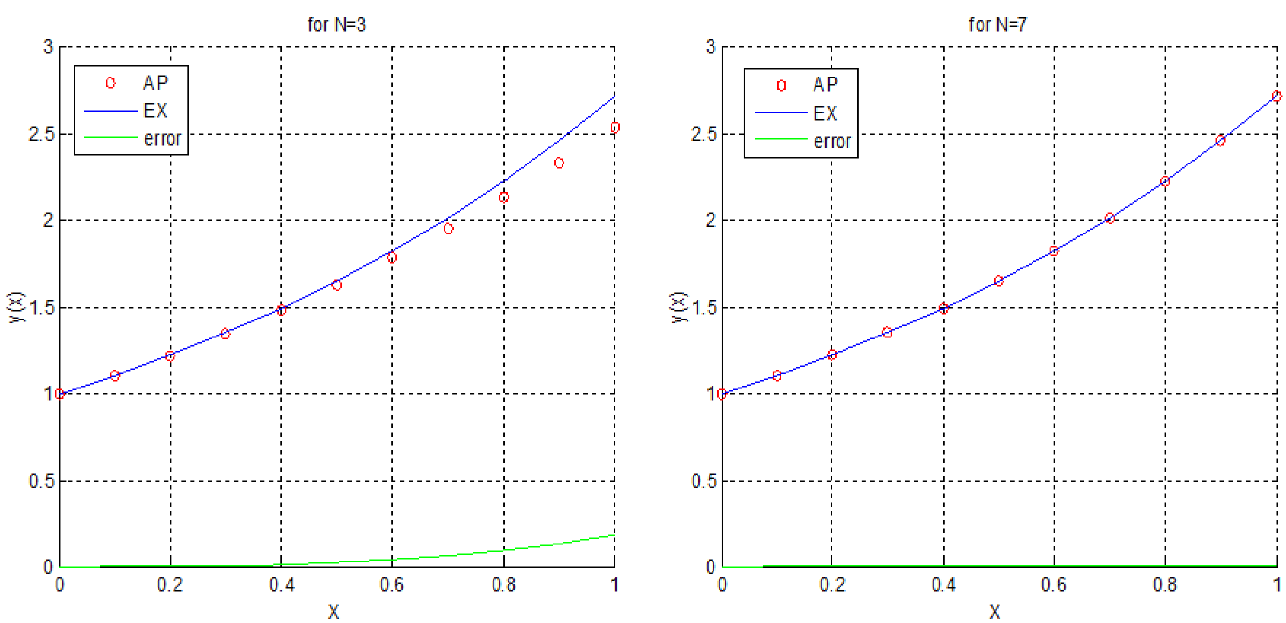
4.1 Example 1

Let us consider the following FIDE :

$$\begin{cases} y'' + xy' - xy = e^x - 2sin(x) + \int_{-1}^{1} sin(x)e^{-t}y(t)dt \qquad -1 \leq x,\ t \leq 1 \\ y(0) = 1 \qquad y'(0) = 1 \end{cases}$$

We mentioned in the Chapter 3 that the exact solution is given by $y_{ex}(x) = e^x$

x	*exact(n=3)*	*approach(n=3)*	*error(n=3)*	*approach(n=7)*	*error(n=7)*
0	*1.0000*	*1.0000*	*0*	*1.0000*	*0*
0.1000	*1.1052*	*1.1046*	*5.7220e-004*	*1.1052*	*1.5710e-008*
0.2000	*1.2214*	*1.2187*	*2.6739e-003*	*1.2214*	*5.4584e-008*
0.3000	*1.3499*	*1.3429*	*6.9673e-003*	*1.3499*	*7.5564e-008*
0.4000	*1.4918*	*1.4776*	*1.4237e-002*	*1.4918*	*3.3868e-008*
0.5000	*1.6487*	*1.6233*	*2.5403e-002*	*1.6487*	*8.5675e-010*
0.6000	*1.8221*	*1.7806*	*4.1534e-002*	*1.8221*	*4.0990e-007*
0.7000	*2.0138*	*1.9499*	*6.3865e-002*	*2.0138*	*2.5302e-006*
0.8000	*2.2255*	*2.1317*	*9.3812e-002*	*2.2255*	*9.1895e-006*
0.9000	*2.4596*	*2.3266*	*1.3299e-001*	*2.4596*	*2.5903e-005*
1.0000	*2.7183*	*2.5350*	*1.8325e-001*	*2.7182*	*6.2489e-005*



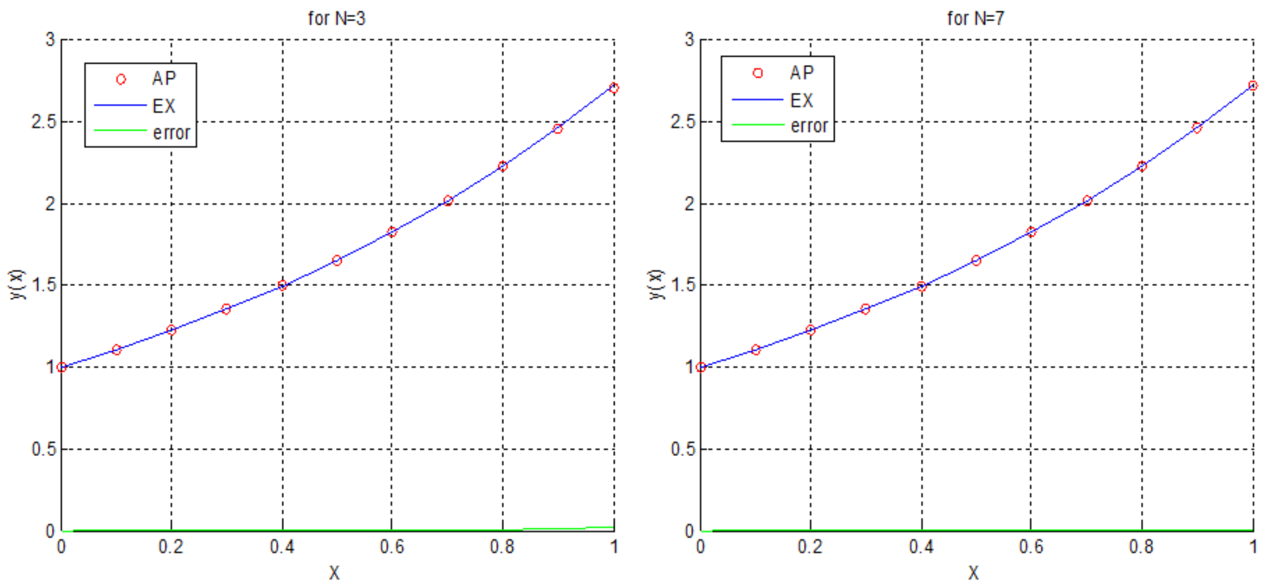
4.2 Example 2

Let us consider the following FIDE

$$\begin{cases} y''(x) = e^x - x + \int_0^1 xty(t)dt; & 0 \leq x,\ t \leq 1 \\ y(0) = 1 \qquad y'(0) = 1 \end{cases}$$

Where the exact solution is given by $y_{ex}(x) = e^x$

x	*exact(n=3)*	*approach(n=3)*	*error(n=3)*	*approach(n=7)*	*error(n=7)*
0	*1.0000*	*1.0000*	*0*	*1.0000*	*0*
0.1000	*1.1052*	*1.1052*	*2.6525e-005*	*1.1052*	*9.7676e-010*
0.2000	*1.2214*	*1.2216*	*1.7679e-004*	*1.2214*	*3.2358e-009*
0.3000	*1.3499*	*1.3503*	*4.7216e-004*	*1.3499*	*4.9497e-009*
0.4000	*1.4918*	*1.4926*	*8.1167e-004*	*1.4918*	*6.7194e-009*
0.5000	*1.6487*	*1.6497*	*9.5913e-004*	*1.6487*	*8.6506e-009*
0.6000	*1.8221*	*1.8226*	*5.2893e-004*	*1.8221*	*9.9418e-009*
0.7000	*2.0138*	*2.0127*	*1.0297e-003*	*2.0138*	*1.2198e-008*
0.8000	*2.2255*	*2.2211*	*4.4500e-003*	*2.2255*	*1.2712e-008*
0.9000	*2.4596*	*2.4489*	*1.0667e-002*	*2.4596*	*4.2589e-008*
1.0000	*2.7183*	*2.6974*	*2.0839e-002*	*2.7183*	*3.9510e-007*



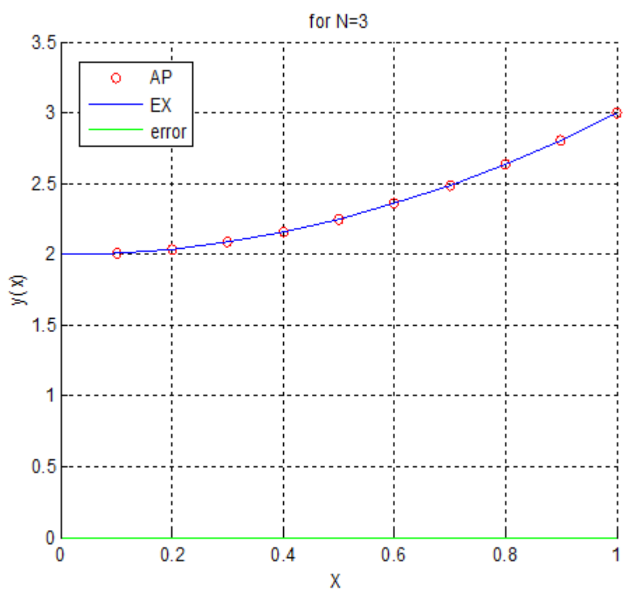
4.3 Example 3

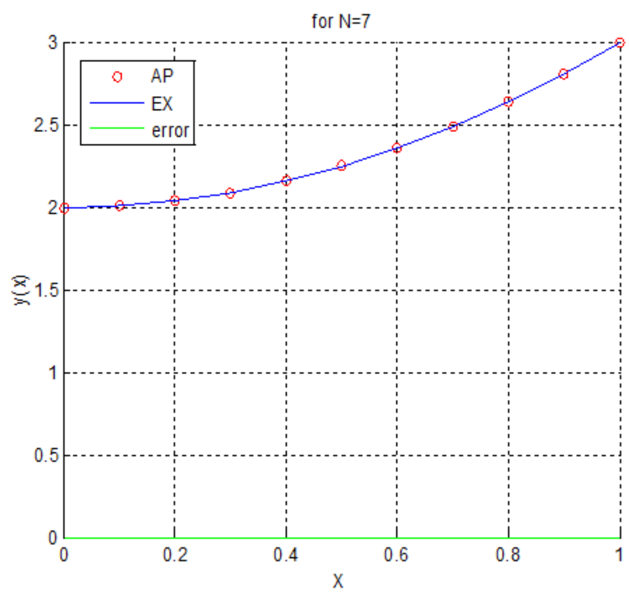
Let us consider the following FIDE

$$\begin{cases} xy'(x)+y(x)=3x^2+\frac{14}{3}x+2-\int_{-1}^{1}(x+t)y(t)dt & 0\leq x\ ,\ t\leq 1 \\ y(0)=2 \end{cases}$$

The exact solution is given by $y(x) = x^2 + 2$

x	exact(n=3)	approach(n=3)	error(n=3)	approach(n=7)	error(n=7)
0	2.0000	2.0000	0	2.0000	0
0.1000	2.0100	2.0100	4.4409e-016	2.0100	4.4409e-016
0.2000	2.0400	2.0400	0	2.0400	0
0.3000	2.0900	2.0900	4.4409e-016	2.0900	4.4409e-016
0.4000	2.1600	2.1600	0	2.1600	0
0.5000	2.2500	2.2500	0	2.2500	4.4409e-016
0.6000	2.3600	2.3600	4.4409e-016	2.3600	4.4409e-016
0.7000	2.4900	2.4900	4.4409e-016	2.4900	8.8818e-016
0.8000	2.6400	2.6400	4.4409e-016	2.6400	4.4409e-016
0.9000	2.8100	2.8100	4.4409e-016	2.8100	1.3323e-015
1.0000	3.0000	3.0000	4.4409e-016	3.0000	6.2172e-015





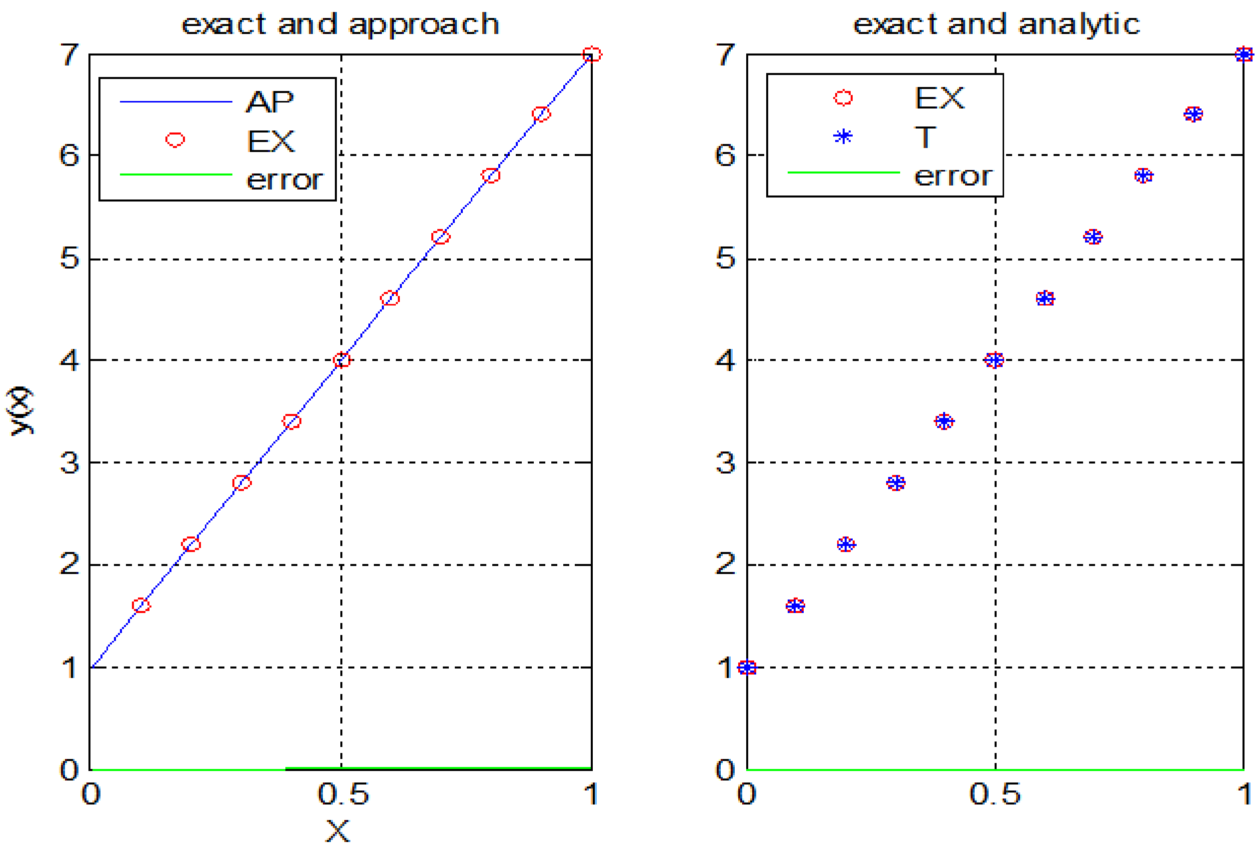
4.4 Example 4

Let us consider the following FIDE :

$$\begin{cases} y'(x) = 4 + 4x + \int_{-1}^{1} (1 - xt) y(t) dt & 0 \leq x \ , \ t \leq 1 \\ y(0) = 1 \end{cases}$$

In this example we will compare between the approach, exact and the analytic solution for n=5

x	*exact*	*approach*	*error*	*analytic*	*error*
0	*1.0000*	*1.0000*	*0*	*1.0000*	*0*
0.1000	*1.6000*	*1.6000*	*0*	*1.6000*	*0*
0.2000	*2.2000*	*2.2000*	*0*	*2.2000*	*0*
0.3000	*2.8000*	*2.8000*	*4.4409e-016*	*2.8000*	*0*
0.4000	*3.4000*	*3.4000*	*1.3323e-015*	*3.4000*	*0*
0.5000	*4.0000*	*4.0000*	*2.6645e-015*	*4.0000*	*0*
0.6000	*4.6000*	*4.6000*	*6.2172e-015*	*4.6000*	*0*
0.7000	*5.2000*	*5.2000*	*1.0658e-014*	*5.2000*	*0*
0.8000	*5.8000*	*5.8000*	*1.8652e-014*	*5.8000*	*0*
0.9000	*6.4000*	*6.4000*	*2.9310e-014*	*6.4000*	*0*
1.0000	*7.0000*	*7.0000*	*4.6185e-014*	*7.0000*	*0*


exact and approach
AP
EX
error
y(x)
X
0
0.5
1
0
1
2
3
4
5
6
7
exact and analytic
EX
T
error

Conclusion

in this thesis we had presented

- *An analaytic method for solving linear FIDE of second kind which is the series solution method*
- *A numerical method called Taylor Collocation method ,where we wrote the solution as a* ***Taylor series***

In the end we proposed some FIDE and tested this method ,

when we compared the exact solution of the example 3 which is a polynom to the approch solution we recognized that they are too close and we found the error almost null

Bibliography

[1] Z.Bi,*2019, Description of Elements, infinte element Analysis application Purdue University Fort Wayne, Indiana, United States*

[2] h.Brezis. *Analyse Functionnelle. Masson, Paris 1983*

[3] H.CHABAH, *2019, Sur les méthodes de Nystrom et applications aux équations intégrales et intégro-differentielles ,université de Batna 2, Mostefa Ben Boulaid*

[4] D.FERAHTIA, *2021, Resolution d'équations intégro-différentielles linéaires de Fredholm-Volterra par une méthode de Collocation,université de Mohamed Boudiaf M'sila*

[5] A.Karamete and M.Sezer, *2002, A Taylor Collocation Method for the Solution of Linear Integro-Differential equations, International Journal of Computer Mathematics, 79:9, 987-1000*

[6] M.MOUSSAI *, 2018, Résolution des équations intégro-différentielles M'sila ,univerité Mohamed Boudiaf*

[7] M. NADIR, *2004, Cours d'analyse fonctionnelle ,université de M'sila, Algérie,*

[8] M.N NADIR, *2022, Notions dÂ"analyse fonctionnelle,Sur la solution numérique des équations intégrales de Volterra-Fredholm en utilisant les polynomes de Chebyshev*

[9] A.WAZWAZ, *2011, Fredholm Integro-Differential Equations,Linear and Nonlinear Integral Equations Methods and Applications , Saint Xavier University ,Chicago*

الملخص : الهدف من هذا العمل هو فرض طريقة لحل معادلة فريدهولم التكاملية التفاضلية الخطية عدديا , تسمى تايلور التجميعية بسلسلة تايلور التقطيعية باستعمال التجميع النقطي لتايلور .

هذه الطريقة مرتكزة على تحويل المعادلة التكاملية التفاضلية إلى معادلة مصفوفة و هذا يقودنا إلى برنامج جبري خطي حيث يسهل علينا حلها . و عرضنا بعض الأمثلة لنبرهن فعالية هذه الطريقة

الكلمات المفتاحية : معادلة فريدهولم التكاملية التفاضلية ,كثير الحدود لتايلور, سلسلة تايلور .

The objective of this work is proposing a method which is Taylor Collocation method for solving linear Fredholm integro-Differential equation numerically, using Taylor collocation points , this method is based on transforming the IDE into a matrix equation which leads to a system of linear algebric equation where we can solve it easly . and we presented some examples to prove the effectiveness of our method

keywords: *integro-differential equations, Taylor polynomials and series, Taylor collocation point..*

L'objectif de ce travail est de proposer une méthode qui est la méthode de collocation de taylor pour résoudre les équations integro-différentielles linéaires numériquement utilisant la méthode des points de collocation de taylor

cette méthode est basée sur la transformation des EIDs en equation matricielle qui mene vers un system d'équation linéaire algebrique , où nous pouvons le resoudre aisement .

Nous avons aussi présenté quelques examples pour preuver l'éfficacité de notre méthode .

Keywords:

les équations integro-differentielle, les polynomes et les séries de taylor, les points de collocation de Taylor